

Math Foundations 10 – Music Monday

Saskatchewan Foundations of Math & Pre-Calculus 10 Curriculum Outcomes reinforced or incorporated in the below content:

FP10.1 Demonstrate understanding of factors of whole numbers by determining the:

- **prime factors**
- **greatest common factor**
- **least common multiple**
- **principal square root**
- **cube root.**

What the following clip on the Fibonacci Sequence in Music

<https://www.youtube.com/watch?v=pOwMDO0-zBw>

The following content has been acquired from:

<http://www.goldennumber.net/music/>

Music and the Fibonacci Sequence and Phi

Musical scales are related to Fibonacci numbers.



The Fibonacci series appears in the foundation of aspects of art, beauty and life. Even music has a foundation in the series, as:

- There are 13 notes in the span of any note through its octave.
- A scale is composed of 8 notes, of which the
- 5th and 3rd notes create the basic foundation of all chords, and
- are based on a tone which are combination of 2 steps and 1 step from the root tone, that is the 1st note of the scale.

Note too how the piano keyboard scale of C to C above of 13 keys has 8 white keys and 5 black keys, split into groups of 3 and 2. While some might “note” that there are only 12 “notes” in the scale, if you don’t have a root and octave, a start and an end, you have no means of calculating the gradations in between, so this 13th note as the octave is essential to computing the frequencies of the other notes. The word “octave” comes from the Latin word for 8, referring to the eight tones of the complete musical scale, which in the key of C are C-D-E-F-G-A-B-C.



In a scale, the dominant note is the 5th note of the major scale, which is also the 8th note of all 13 notes that comprise the octave. This provides an added instance of Fibonacci numbers in key musical relationships. Interestingly, 8/13 is .61538, which approximates phi. What's more, the typical three chord song in the key of A is made up of A, its Fibonacci & phi partner E, and D, to which A bears the same relationship as E does to A. This is analogous to the "A is to B as B is to C" basis for the golden section, or in this case "D is to A as A is to E."

Musical frequencies are based on Fibonacci ratios

Notes in the scale of western music are based on natural harmonics that are created by ratios of frequencies. Ratios found in the first seven numbers of the Fibonacci series (0, 1, 1, 2, 3, 5, 8) are related to key frequencies of musical notes.

Fibonacci Ratio	Calculated Frequency	Tempered Frequency	Note in Scale	Musical Relationship	When A=432 *	Octave below	Octave above
1/1	440	440.00	A	Root	432	216	864
2/1	880	880.00	A	Octave	864	432	1728
2/3	293.33	293.66	D	Fourth	288	144	576
2/5	176	174.62	F	Aug Fifth	172.8	86.4	345.6
3/2	660	659.26	E	Fifth	648	324	1296
3/5	264	261.63	C	Minor Third	259.2	129.6	518.4
3/8	165	164.82	E	Fifth	162 (Phi)	81	324
5/2	1,100.00	1,108.72	C#	Third	1080	540	2160
5/3	733.33	740.00	F#	Sixth	720	360	1440
5/8	275	277.18	C#	Third	270	135	540
8/3	1,173.33	1,174.64	D	Fourth	1152	576	2304
8/5	704	698.46	F	Aug. Fifth	691.2	345.6	1382.4

The calculated frequency above starts with A440 and applies the Fibonacci relationships. In practice, pianos are tuned to a "tempered" frequency, a man-made adaptation devised to provide

improved tonality when playing in various keys. Pluck a string on a guitar, however, and search for the harmonics by lightly touching the string without making it touch the frets and you will find pure Fibonacci relationships.

* A440 is an arbitrary standard. The American Federation of Musicians accepted the A440 as standard pitch in 1917. It was then accepted by the U.S. government its standard in 1920 and it was not until 1939 that this pitch was accepted internationally. Before recent times a variety of tunings were used. It has been suggested by James Furia and others that A432 be the standard. A432 was often used by classical composers and results in a tuning of the whole number frequencies that are connected to numbers used in the construction of a variety of ancient works and sacred sites, such as the Great Pyramid of Egypt. The controversy over tuning still rages, with proponents of A432 or C256 as being more natural tunings than the current standard.

Musical compositions often reflect Fibonacci numbers and phi

Fibonacci and phi relationships are often found in the timing of musical compositions. As an example, the climax of songs is often found at roughly the phi point (61.8%) of the song, as opposed to the middle or end of the song. In a 32 bar song, this would occur in the 20th bar.

Musical instrument design is often based on phi, the golden ratio



Fibonacci and phi are used in the design of violins and even in the design of high quality speaker wire.

Insight on Fibonacci relationship to dominant 5th in major scale contributed by Sheila Yurick.